Sastavila: Olga Nerlović
Dulce cum utili. Ugodno s korisnim.

IV. 1. Funkcije. Vježba. Viša razina

1. Odredi prirodno područje definicije funkcije:

a) $f(x)=\sqrt{x-x^3}$ $x\in\langle-\infty;-1]\cup[0;1]$

b) $f(x)=\frac{2}{x^3-x}$ $x\in R\setminus\{-1;0;1\}$

c) $f(x)=\sqrt[4]{|x|-2x^2}$ $x\in\left[-\frac{1}{2};\frac{1}{2}\right]$

d) $f(x)=\sqrt{x-1}+\sqrt[3]{x+1}$ $x\in[1;+\infty\rangle$

e) $f(x)=\log(2+x-x^2)$ $x\in\langle-1;2\rangle$

f) $f(x)=\frac{1}{\left(\frac{1}{2}\right)^x-2}$ $x\in R\setminus\{-1\}$

g) $f(x)=\sqrt{3^{x-2}-5^{x+1}}$ $x\in\langle-\infty;\frac{\log 9-\log 5}{\log 5-\log 3}]$

h) $f(x)=\sqrt{\frac{x^2}{x^2-4}}$ $x\in\langle-\infty;-2\rangle\cup\{0\}\cup\langle2;+\infty\rangle$

i) $f(x)=\sqrt{\frac{(x-2)^2}{x^2-25}}$ $x\in\langle-\infty;-5\rangle\cup\{2\}\cup\langle5;+\infty\rangle$

j) $f(x)=\frac{\sqrt{35-2x-x^2}}{\log x}$ $x\in\langle0;1\rangle\cup\langle1;5]$

k) $f(x)=\frac{\log(-x^2+3x+4)}{\sqrt{x-2}}$ $x\in\langle2;4\rangle$

l) $f(x)=\sqrt{3^{2x}-2\cdot3^x-3}$ $x\in[1;+\infty\rangle$

m) $f(x)=\frac{x}{\sin x}$ $x\in R\setminus\{x=k\pi,k\in Z\}$

n) $f(x)=\sqrt{\log\frac{x+8}{x-1}-1}$ $x\in\langle1;2]$

2. Odredite prirodno područje definicije funkcije:

a) $f(x)=\frac{1}{\sqrt{2}\sin x-1}$ $x\in R\setminus\left\{x=\frac{\pi}{4}+2k\pi,x=\frac{3\pi}{4}+2k\pi,k\in Z\right\}$

b) $f(x)=\sqrt{\frac{2}{\sqrt{3}}\cos x-1}$ $x\in\left[-\frac{\pi}{6}+2\pi k;\ \frac{\pi}{6}+2\pi k.\,k\in Z\right]$

c) $f(x)=\frac{\sin x}{\cos x}$ $x\in R\setminus\left\{x=\frac{\pi}{2}+k\pi,k\in Z\right\}$

d) $f(x)=\frac{1}{tgx}$ $x\in R\setminus\left\{\frac{\pi}{2}k,k\in Z\right\}$

e) $f(x)=\log(\sin x+2)$ $x\in R$

f) $f(x)=\log_{0.5}(1+\cos x)$ $x\in R\setminus\{x=\pi+2k\pi,k\in Z\}$

3. Zadana je funkcija $f(x)=x^2$. Odredi $f(0),\ f(-x),f(a+1),f(2a)$

R: $0,x^2.(a+1)^2.4a^2$

4. Odredi $f(0),f(-x),\ f(x+1),\ f(x)+1.\ f\left(\frac{1}{x}\right),\ \frac{1}{f(x)}$ ako je $f(x)=\frac{1-x}{1+x}$

R: $1;\frac{1+x}{1-x};\frac{-x}{2+x};\frac{2}{1+x};\frac{x-1}{x+1};\frac{1+x}{1-x}$

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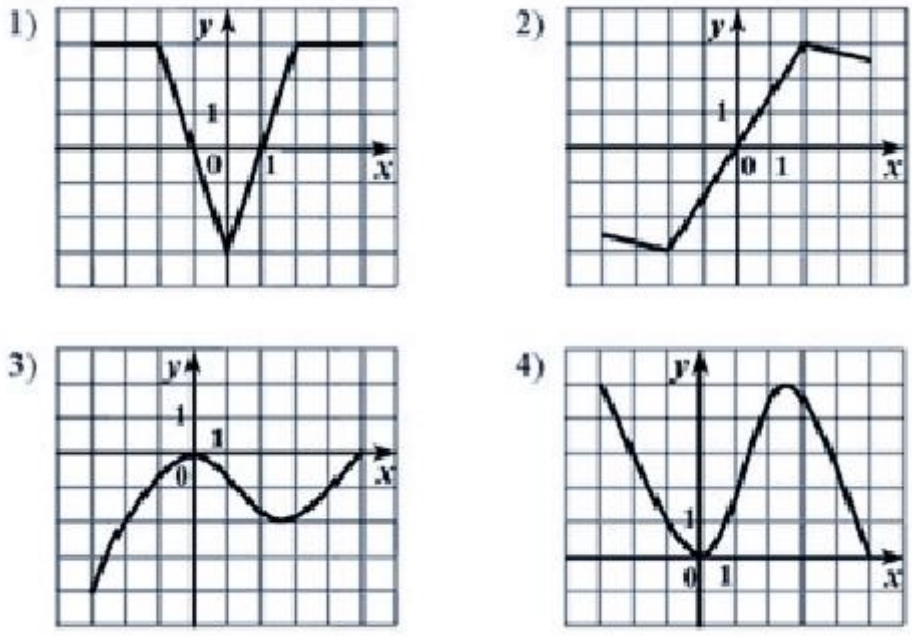
5. Na kojem od navedenih grafova predstavljena parna funkcija:

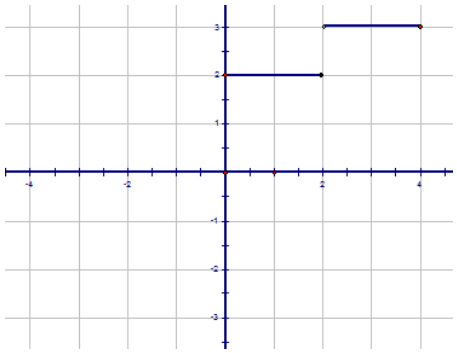


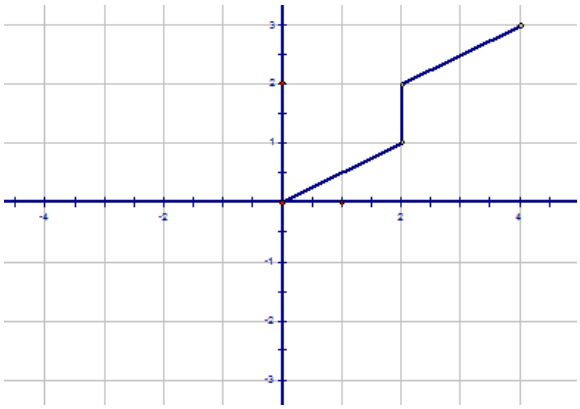
(1)

6. Ispitaj funkcije na parnost i neparnost:
   a) $f(x) = x^2 - x$
   b) $f(x) = 1 - \frac{x}{2}$
   c) $f(x) = -\frac{3}{x}$
   d) $f(x) = x^2 + 2x + 1$
   e) $f(x) = x^4 + 5x^2$
   f) $f(x) = \left(\frac{1}{2}\right)^{-x}$
   g) $f(x) = x^3 + \sin x$
   h) $f(x) = x^2 \cos x$
   i) $f(x) = x \cos x$
   j) $f(x) = \sin x - \cos x$

R:
a) Niti parna, niti neparna
b) Niti parna, niti neparna
c) Neparna
d) Niti parna, niti neparna
e) Parna
f) Niti parna, niti neparna
g) Neparna
h) Parna
i) Neparna
j) Niti parna, niti neparna

7. Docrtaj grafovi funkcija tako da budu parne ili neparne:

a) 

b) 

8. Koji od sljedećih brojeva -2, -1, 0, 1, 2 pripadaju prirodnomu području definicije (domeni) funkcije:
   a) $y = f(f(x))$
   b) $y = g(g(x))$
   c) $y = f \circ g(x)$
   d) $y = g \circ f(x)$

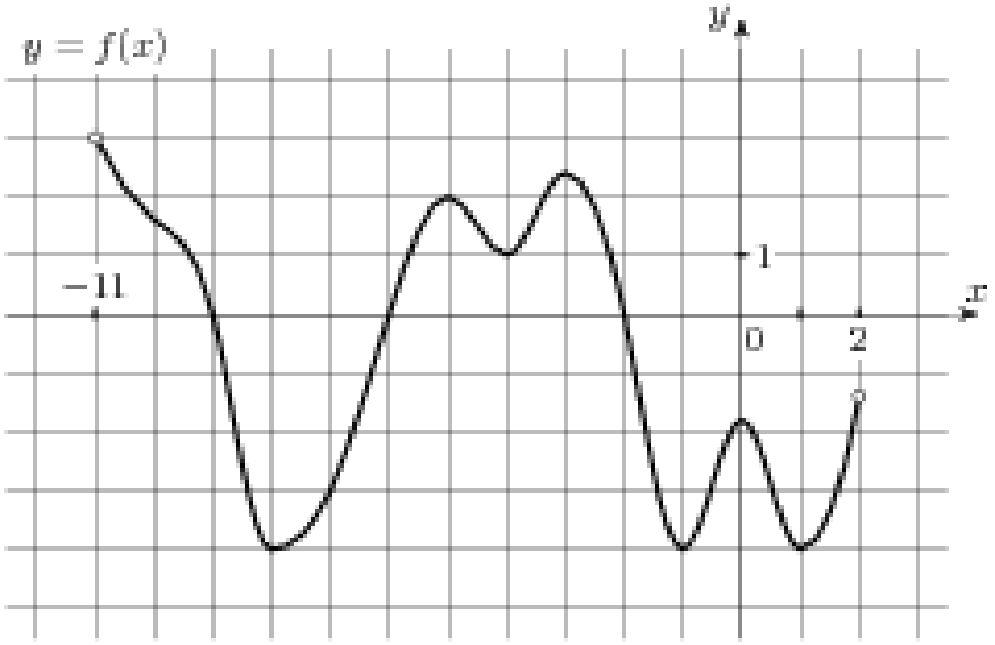
   Ako le $f(x) = \sqrt{x}$ i $g(x) = x^4 - 4$ te odredite odgovarajući vrijednosti tim brojevima.

R: a) 0, 1, 2..; b)… ,c) -2;2, d) …

9. Predočite funkciju $y = F(x)$ kao složenu funkciju jednostavnijih funkcija f i g:
   a) $F(x) = (3x + 4)^2$
   b) $F(x) = \sqrt{2x + 1}$
   c) $F(x)\cos(5x + 2)$
   d) $F(x) = \log(x^2 + 4x + 1)$

R: a) $f(x) = x^2;\ g(x) = 3x + 4$
b) $f(x) = \sqrt{x}\,; g(x) = 2x + 1$
c) $f(x) = \cos x\,;\ g(x) = 5x + 2$
d) $f(x) = \log x\,;\ g(x) = x^2 + 4x + 1$

10. Zadani su funkcije slika 1) i 2) odredite:
    a) Prirodno područje definicije
    b) Područje vrijednosti
    c) Nultočke
    d) Intervali rasta
    e) Intervali pada
    f) Intervali gdje je funkcija niti rastuća niti padajuća
    g) Maksimum i minimum funkcije
    h) Pozitivna; negativna



R:

a)$x \in \langle -11; 2\rangle$

b) $y \in [-4; 3\rangle$

c) $x_2 = -9; x_2 = -6; x_3 = -2$

d) $x \in \langle -8; -5\rangle \cup \langle -4; -3\rangle \cup \langle -1; 0\rangle \cup \langle 1; 2\rangle$

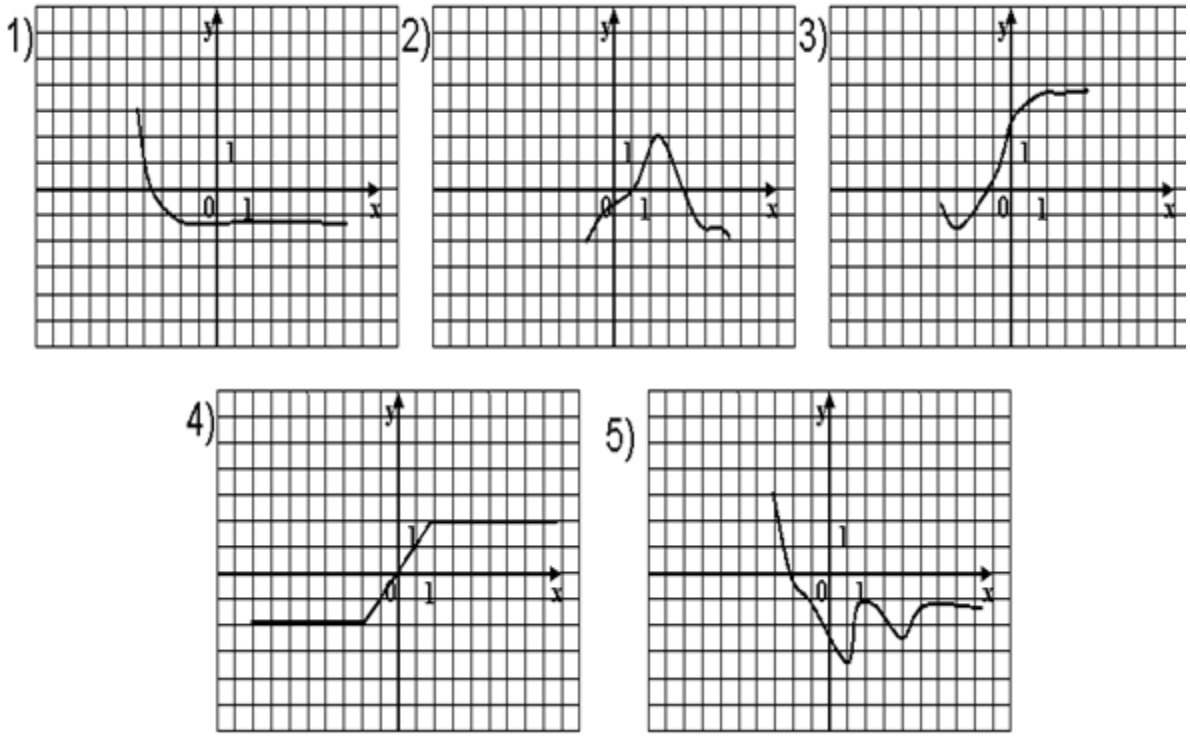
e) $x \in \langle -11; -8\rangle \cup \langle -5; -4\rangle \cup \langle -3; -1\rangle \cup \langle 0; 1\rangle$

f) $x \in \{\emptyset\}$

g)$M(-5; 2); M(-3; 2.5); M(0; -2)$;

$m(-8; -4); m(-4; 1); m(-1; -4)m(1; -4)$

11. Odaberi crtež koji predstavlja funkciju koja na intervalu (-2,2) poprima samo negativne vrijednosti.



12. Nacrtajte grafovi funkcija:

a) $f(x) = x^{\log_x(4x-2)}$

b) $f(x) = 3^x + \log_x 1$

c) $f(x) = \log_x(4-x) \cdot \log_{(4-x)} x$

d) $f(x) = (x^4)^{\log_{x^4}|x|}$

e) $f(x) = \frac{\log x^6}{\log x}$

f) $f(x) = 7^{\log_7(x^2-2x-3)}$

13. Koliko rješenja ima jednadžba $(x+2)^2 = -\frac{3}{x}$?

A: 0 B: 1 C: 2 D: 4 (B)

14. Koliko rješenja ima jednadžba $3^{-x} = 4 + x - x^2$?

A: 0; B: 1; C: 2; D: 3 (C)

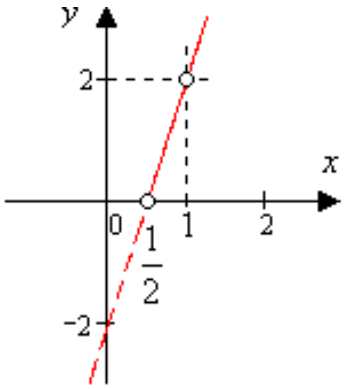
15. Udaljenost između ishodišta koordinatnog sustava i točki presjeka grafova funkcija $f(x) = x^2$ *i* $g(x) = \frac{1}{x}$ iznosi:

A: 1 B: $\sqrt{2}$ C: 0 D: 2 (B)

Rješenje zadatka 12.

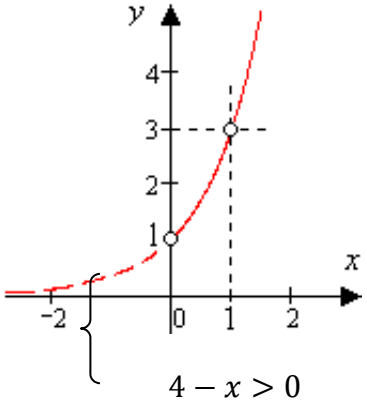
a) D(f): $\begin{cases} 4x-2>0 \\ x>0 \\ x \neq 1 \end{cases}$ => $D(f) = \langle \frac{1}{2}; 1\rangle \cup \langle 1; +\infty\rangle$

$$f(x) = x^{\log_x(4x-2)} = 4x - 2$$



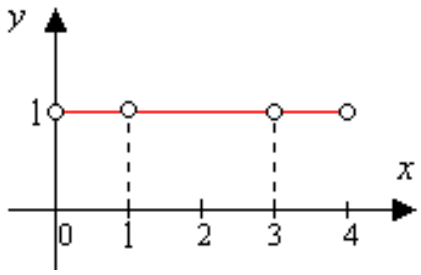
b) D(f): $\begin{cases} x > 0 \\ x \neq 1 \end{cases}$ => $D(f) = \langle 0; 1\rangle \cup \langle 1;\ +\infty\rangle$

$f(x) = 3^x + \log_x 1 = 3^x + 0 = 3^x$



c) D(f): $\begin{matrix} 4 - x > 0 \\ x > 0 \\ x \neq 1;\quad 4 - x \neq 1 \end{matrix}$ => $D(f) = \langle 0; 1\rangle \cup \langle 1; 3\rangle \cup \langle 3; 4\rangle$

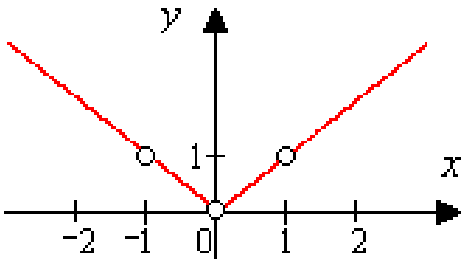
$f(x) \log_x(4 - x) \cdot \log_{(4-x)} x = 1$



d) D(f): $\begin{cases} |x| > 0 \\ x^4 > 0 \\ x^4 \neq 1 \end{cases}$ => $[x \neq \pm 1, x \neq 0]$ =>

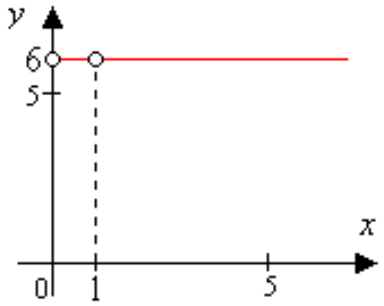
$D(f) = \langle -\infty; -1\rangle \cup \langle -1; 0\rangle \cup \langle 0; 1\rangle \cup \langle 1; ; +\infty\rangle\ ili\ x \in R \setminus \{-1; 1;\ 0\}$

$f(x) = (x^4)^{\log_{x^4}|x|} = |x|$



e) D(f): $\begin{cases} x^6 > 0 \\ x > 0 \\ \log x \neq 0 \end{cases}$ => $D(f) = \langle 0; 1\rangle \cup \langle 1; +\infty\rangle$

$$f(x) = \frac{\log x^6}{\log x} = \frac{6\log|x|}{\log x} = \frac{6\log x}{\log x} = 6$$



f) D(f): $x^2 - 2x - 3 > 0$ => $D(x) = \langle -\infty; -1\rangle \cup \langle 3; +\infty\rangle$

$f(x) = 7^{\log_7(x^2-2x-3)} = x^2 - 2x - 3$