

## II. 5. Trigonometrijske funkcije. Trigonometrijske jednačbe i nejednačbe

## Kompleksni brojevi u trigonometrijskom obliku Vježba. Viša razina

1. Izračunajte:

a)  $\sin\left(\frac{77\pi}{6}\right) \cdot \operatorname{ctg}\left(-\frac{53\pi}{3}\right);$

c)  $\frac{\cos x}{1-\sin x} + \frac{1-\sin x}{\cos x}$  za  $x = -\frac{23\pi}{3}$

b)  $\cos\left(-\frac{100\pi}{3}\right) \cdot \operatorname{tg}\left(\frac{35\pi}{6}\right)$

d)  $\sin\frac{5\pi}{6} \cos\frac{2\pi}{3} + \cos\frac{5\pi}{6} \sin\frac{2\pi}{3}$

R: a)  $\frac{\sqrt{3}}{6};$  b)  $\frac{\sqrt{3}}{6};$  c) -4; d) -1

2. Izračunaj vrijednost ostalih trigonometrijskih funkcija ako je zadano:

a)  $\sin t = \frac{4}{5}, t \in \left(\frac{\pi}{2}; \pi\right)$   $\left(\cos t = -\frac{3}{5} \dots\right)$

b)  $\operatorname{tg} t = -0.75, t \in \left(\frac{3\pi}{2}; 2\pi\right)$   $\left(\cos t = \frac{4}{5} \dots\right)$

3. Ako je  $\sin x = \frac{5\sqrt{3}}{14}$  i  $\cos y = -\frac{3\sqrt{3}}{14}$ ,  $x \in \left(0; \frac{\pi}{2}\right)$ ,  $y \in \left(\frac{\pi}{2}; \pi\right)$  koliko je

a)  $\sin(x+y);$

c)  $\sin 2x$

b)  $\cos(x-y)$

d)  $\cos 2x$

R: a)  $\frac{1}{2};$  b)  $\frac{8\sqrt{3}}{49};$  c)  $\frac{55\sqrt{3}}{98};$  d)  $\frac{23}{98}$

4. Koliko je  $\operatorname{tg} x + \operatorname{ctg} x$  ako je  $\sin x \cdot \cos x = m$ 

$\left(\frac{1}{m}\right)$

5. Pojednostavi:

a)  $\frac{1-\cos^2 t}{\sin t \cos t};$   $(\operatorname{tg} x)$

b)  $\frac{1+\cos t}{\sin t} - \frac{\sin t}{1+\cos t}$   $(2\operatorname{ctg} x)$

c)  $\sin^2\left(\frac{5\pi}{2} - x\right) - \sin^2\left(\frac{5\pi}{2} + x\right)$   $(0)$

6. Izračunaj  $\cos\left(\frac{\pi}{3} - x\right)$  ako je  $\cos x = \frac{5}{13}, x \in \left(\frac{3\pi}{2}; 2\pi\right)$   $\left(\frac{5-12\sqrt{3}}{26}\right)$ 

7. Riješi jednačbe:

a)  $\cos x = -\frac{1}{2}$   $\left(x_1 = \frac{2\pi}{3} + 2\pi k; x_2 = \frac{4\pi}{3} + 2\pi k, k \in \mathbb{Z}\right)$

b)  $\sin x = \frac{\sqrt{2}}{2}$   $\left(x_1 = \frac{\pi}{4} + 2\pi k; x_2 = \frac{3\pi}{4} + 2\pi k, k \in \mathbb{Z}\right)$

c)  $\operatorname{tg} x = \sqrt{3}$   $\left(x = \frac{\pi}{3} + \pi k, k \in \mathbb{Z}\right)$

d)  $\operatorname{ctg} x = -1$   $\left(x = \frac{3\pi}{4} + \pi k, k \in \mathbb{Z}\right)$

e)  $\cos\left(x - \frac{\pi}{2}\right) = \frac{\sqrt{3}}{2}$   $\left(x_1 = \frac{\pi}{3} + 2\pi k; x_2 = \frac{2\pi}{3} + 2\pi k, k \in \mathbb{Z}\right)$

f)  $\sin\left(2x + \frac{\pi}{6}\right) = -1$   $\left(x = \frac{2\pi}{3} + \pi k, k \in \mathbb{Z}\right)$

g)  $2 \sin 3x + 1 = 0$   $\left(x_1 = \frac{7\pi}{18} + \frac{2\pi k}{3}; x_2 = \frac{11\pi}{18} + \frac{2\pi k}{3}, k \in \mathbb{Z}\right)$

8. Riješi jednačbe:

a)  $4\sin^2 x - 1 = 0$   $\left(x = \pm \frac{\pi}{6} + \pi k, k \in \mathbb{Z}\right)$

b)  $2\cos^2 x - \cos x - 1 = 0$   $\left(x_1 = \frac{2\pi}{3} + 2\pi k; x_2 = \frac{4\pi}{3} + 2\pi k, x_3 = 2\pi k, k \in \mathbb{Z}\right)$

c)  $6\operatorname{tg}^2 x - \operatorname{tg} x - 1 = 0$

$\left(x_1 = 26^\circ 33' 54'' + 180^\circ k; x_2 = 161^\circ 33' 54'' + 180^\circ k, k \in \mathbb{Z}\right)$

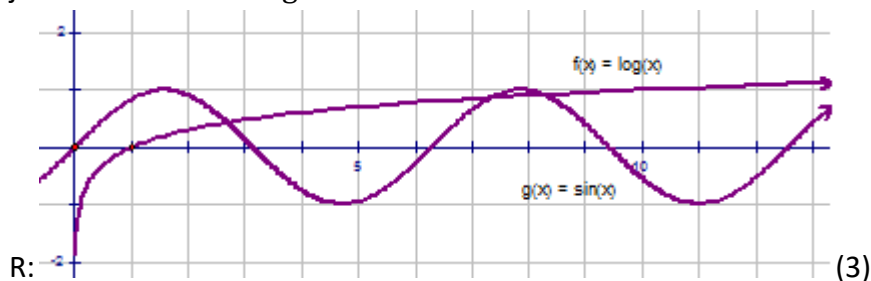
9. Riješi jednađbe:

- a)  $4\cos^2 x - 7 \sin x \cos x + 3\sin^2 x = 0$   $(x_1 = \frac{\pi}{4} + \pi k, x_2 = k \in Z)$   
 b)  $5 \cos 2x = 3 \cos^2 x + \sin^2 x$   $(x = \pm \frac{\pi}{6} + \pi k, k \in Z)$   
 c)  $3 \sin x + \sqrt{3} \cos x = 0$   $(x = \frac{5\pi}{6} + \pi k, k \in Z)$

10. Riješi nejednađbe:

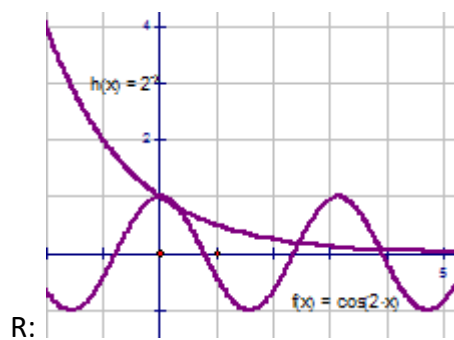
- a)  $\sin x > \frac{1}{2}$   $(x \in (\frac{\pi}{6} + 2\pi k; \frac{5\pi}{6} + 2\pi k), k \in Z)$   
 b)  $\cos x \geq -\frac{\sqrt{2}}{2}$   $(x \in [-\frac{3\pi}{4} + 2\pi k; \frac{3\pi}{4} + 2\pi k], k \in Z)$   
 c)  $2\cos^2 x - 1 < 0$   $(x \in (\frac{\pi}{4} + \pi k; \frac{3\pi}{4} + \pi k), k \in Z)$   
 d)  $2\sin^2 x - \sin x - 1 \geq 0$   $(x \in [\frac{7\pi}{6} + 2\pi k; \frac{11\pi}{6} + 2\pi k] \cup \{\frac{\pi}{2} + 2\pi k\}, k \in Z)$

11. Koliko ima rješenja jednađba  $\sin x = \log x$



12. Broj rješenja jednađba  $\cos 2x = 2^{-x}$  jednak je:

- A: 2;      B: 4;      C: 8;      D: beskonačan (D)



13. Ako je  $x = \frac{3\pi}{4}$  jedno rješenje jednađbe  $\sin^2 x + a \sin x \cos x + \cos^2 x = 0$ , onda je:

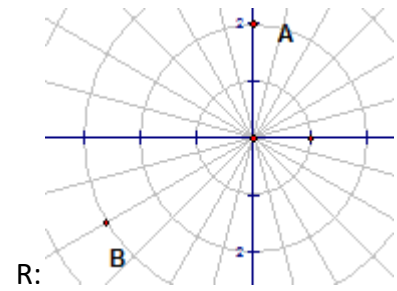
- A:  $a = -1$ ;      B:  $a = 2$ ;      C:  $a = \frac{1}{2}$ ;      D:  $a = 1$  (B)

14. Prikaži u Gaussovoj ravnini sljedeće brojeve:

- a)  $z = 2 \left( \cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right)$   
 b)  $z = 3 \left( \cos \frac{7\pi}{6} + i \sin \frac{7\pi}{6} \right)$

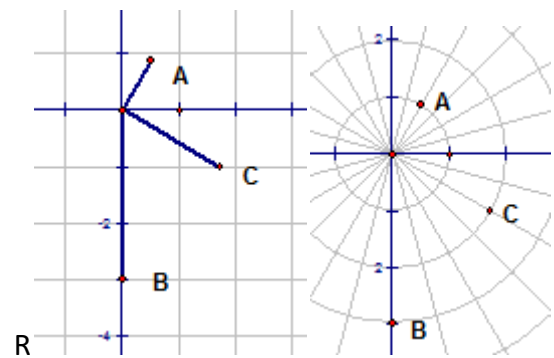
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15. Odredi i nacrtaj točke zadane polarnim koordinatama  $r$  i  $\varphi$ :

- a)  $A\left(1; \frac{\pi}{3}\right)$
- b)  $B\left(3; \frac{3\pi}{2}\right)$
- c)  $C\left(2; \frac{11\pi}{6}\right)$



16. Odredi argumente kompleksnih brojeva i zapiši u trigonometrijskom obliku:

- a)  $z = -1 + i$
- b)  $z = -1 - i\sqrt{3}$
- c)  $z = -\frac{1}{2} + 3i$
- d)  $z = \sqrt{3} + i$
- e)  $z = -\frac{1}{2}i + \frac{\sqrt{3}}{2}$

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R: a)  $\frac{3\pi}{4}$ ; b)  $\frac{4\pi}{3}$ ; c)  $99^\circ 27' 45''$ ; d)  $\frac{\pi}{6}$ ; e)  $\frac{11\pi}{6}$

17. Kompleksne brojeve zapiši u trigonometrijskom obliku:

a)  $z = 2 \cos \frac{7\pi}{4} - 2i \sin \frac{\pi}{4}$

b)  $z = -\cos \frac{\pi}{17} + i \sin \frac{\pi}{17}$

R: a)  $z = 2 \left( \cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} \right)$ ; b)  $z = \cos \frac{16\pi}{17} + i \sin \frac{16\pi}{17}$

18. Odredi umnožak i kvocijent brojeva:

$z_1 = \sqrt{3} \left( \cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$  i  $z_2 = \sqrt{2} \left( \cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$

R:  $z_1 \cdot z_2 = \sqrt{6} \left( \cos \frac{7\pi}{12} + i \sin \frac{7\pi}{12} \right)$ ;  $\frac{z_1}{z_2} = \frac{\sqrt{6}}{2} \left( \cos \frac{\pi}{12} + i \sin \frac{\pi}{12} \right)$

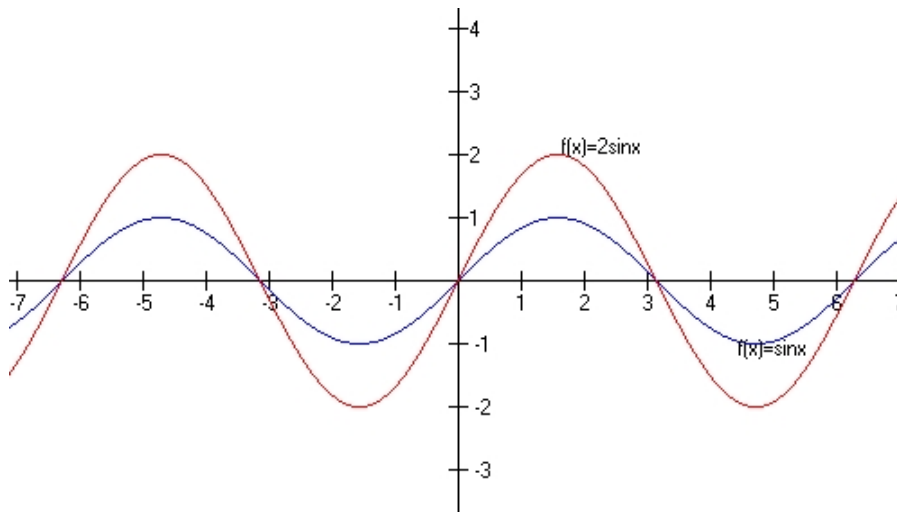
19. Izračunaj:

a)  $(i - \sqrt{3})^{12}$   $(2^{12})$

b)  $(1 - i)^{20}$   $(-2^{10})$

c)  $\sqrt[3]{-1}$   $(z_1 = \frac{1}{2} + \frac{\sqrt{3}}{2}i; z_2 = -1; z_3 = \frac{1}{2} - \frac{\sqrt{3}}{2}i)$

● primjer 1.  $f(x) = 2 \sin x$  - amplituda  $a = 2$

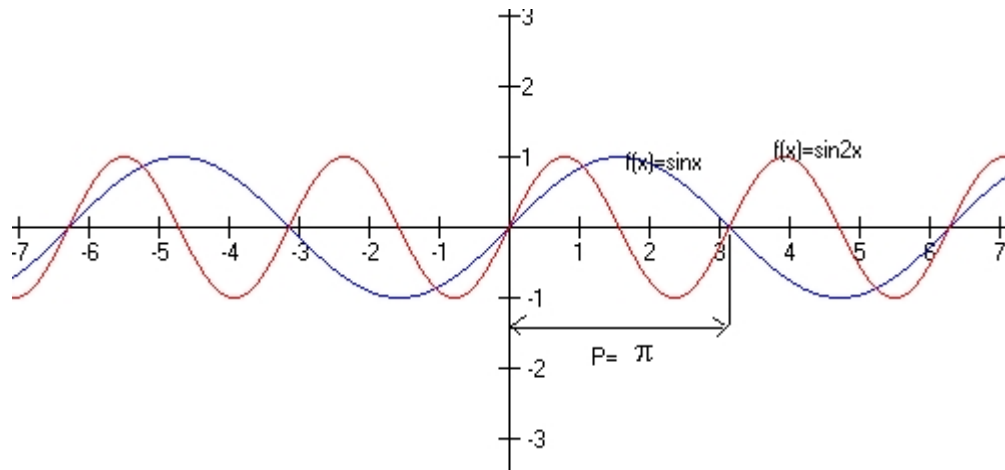


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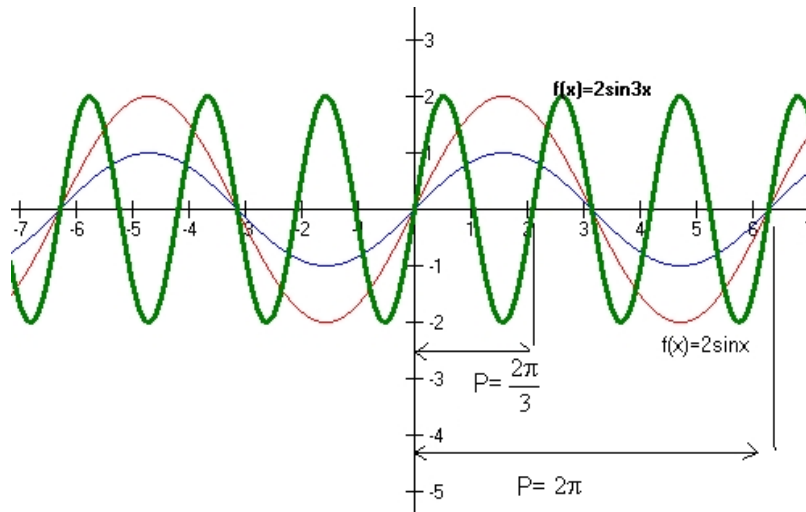
● primjer 2.  $f(x) = \sin 2x$  period

$$P = \frac{2\pi}{|b|} = \frac{2\pi}{2} = \pi$$



● primjer 3.  $f(x) = 2\sin 3x$  period

$$P = \frac{2\pi}{|b|} = \frac{2\pi}{3}, \text{ amplituda } a = 2$$

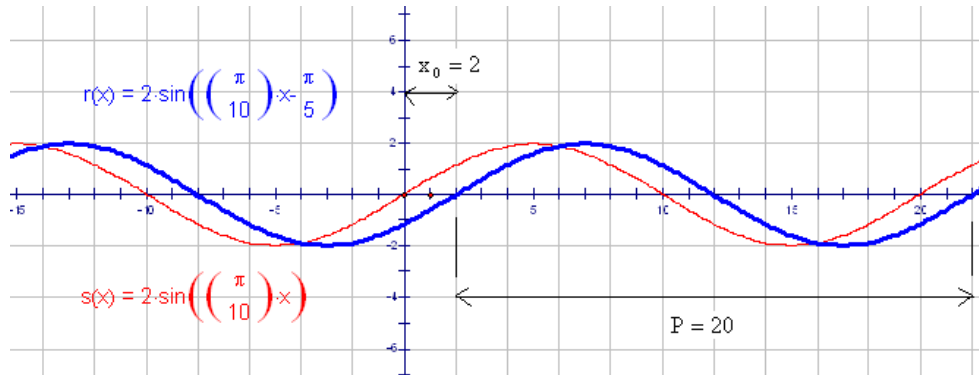


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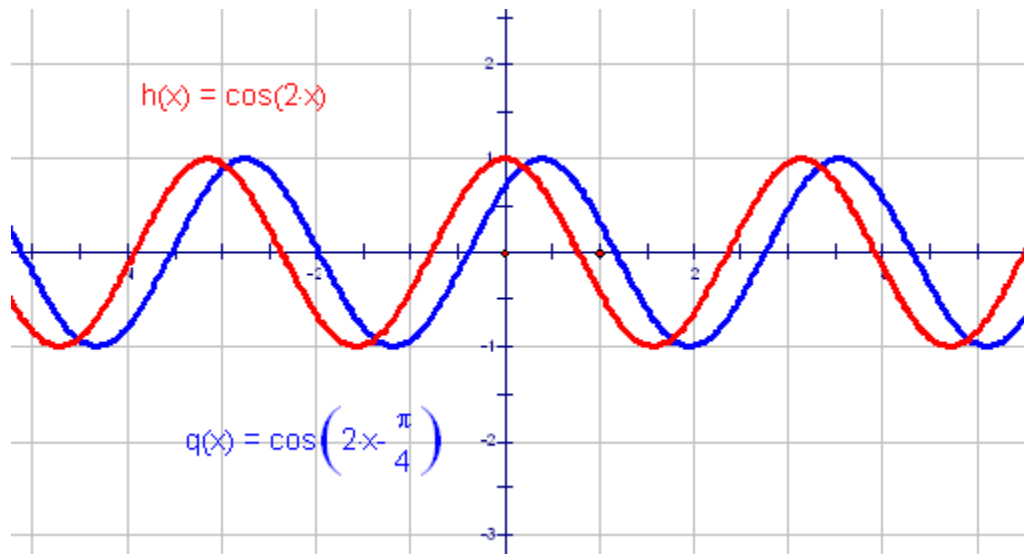
● primjer 4.  $f(x) = 2\sin\left(\frac{\pi}{10}x - \frac{\pi}{5}\right)$

amplituda  $a = 2$ , period funkcije  $P = \frac{2\pi}{|b|} = \frac{2\pi}{\frac{\pi}{10}} = 20$ , pomak  $x_0 = -\frac{c}{b} = -\frac{-\frac{\pi}{5}}{\frac{\pi}{10}} = 2$



● primjer 5.  $f(x) = \cos\left(2x - \frac{\pi}{4}\right) \rightarrow f(x) = \cos\left(2x - \frac{\pi}{4}\right) = \sin\left(2x - \frac{\pi}{4} + \frac{\pi}{2}\right) = \sin\left(2x + \frac{\pi}{4}\right)$

period funkcije  $P = \frac{2\pi}{|b|} = \frac{2\pi}{2} = \pi$ , pomak  $x_0 = -\frac{c}{b} = -\frac{\frac{\pi}{4}}{2} = -\frac{\pi}{8}$



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