Sastavil: Olga Nerlović

Homo doctus in se semper divitias habet. Učen čovijek ima uvijek u sebi bogatstvo.

II. 4. Logaritamska funkcija. Logaritamske jednadžbe i nejednadžbe. Vježba

Viša razina

1. Riješi jednadžbe:
   a) $5^{x+1} + 3^{x-1} = 5^{x-1} + 3^{x+1}$ $(x \approx -1.15)$
   b) $4^x - 5 \cdot 6^x + 4 \cdot 9^x = 0$ $(x_1 = 0, x_2 \approx -3.419)$

2. Izračunaj:

1) $\frac{\log_9 2}{\log_{81} 2}$

2) $\frac{78}{5^{\log_5 6}}$

3) $49^{\log_7 8}$

4) $16^{\log_4 3}$

5) $\frac{65}{9^{\log_9 5}}$

6) $9 \cdot 9^{\log_9 6}$

7) $6 \cdot 7^{\log_7 2}$

8) $5^{\log_{25} 36}$

9) $5 \cdot 11^{\log_{11} 6}$

10) $\log_{11} 24.2 + \log_{11} 5$

11) $\frac{24}{3^{\log_3 2}}$

12) $\log_5 7 \cdot \log_7 25$

13) $\frac{\log_9 \sqrt[5]{17}}{\log_9 17}$

14) $5^{2\sqrt{3}-4} \cdot 25^{1-\sqrt{3}}$

R:

1) 2;
2) 13;
3) 64;
4) 9;
5)13;
6) 54;
7)12;
8)6;
9)30;
10) 2;
11) 12;
12)2;
13) $\frac{1}{5}$;
14) $\frac{1}{25}$

3. Izračunaj vrijednost izraza:

1) $\log_a(a^2b^3)$ $ako\ je$ $\log_b a = \frac{1}{12}$

2) $\log_a \frac{a^9}{b^8}$ $ako\ je$ $\log_a b = 11$

3) $\log_a(a^7b^{10})$ $ako\ je$ $\log_b a = \frac{1}{2}$

4) $\log_a \frac{a}{b^5}$ $ako\ je$ $\log_a b = 13$

5) $\log_a \frac{a}{b^5}$ $ako\ je$ $\log_a b = -4$

R:

1) 38; 2) -79; 3) 27; 4)-64; 5) 21

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4. Skrati razlomke:

   a) $\frac{log^2 x}{\log x^2}$; $\left(\frac{1}{2}\log x\right)$

   b) $\frac{1-(\log b)^2}{log(10b)}$ $(1-\log b)$

5. Riješi jednadžbe:

   a) $\frac{\log(2x-1)}{\log(x^2+5)} = \frac{1}{2}$ (2)

   b) $\log(x-2) + \log(x+2) = 2\log(x-2)$ $(x \in \{\emptyset\})$

   c) $\log(x-5)^2 + \log(x+6)^2 = 2$ $\left(x_1 = -5, x_2 = 4, x_{3,4} = \frac{1\pm\sqrt{161}}{2}\right)$

6. Riješi jednadžbe:

   a) $\log[\log(\log x)] = 0$ $(x = 10^{10})$

   b) $\log[3 + 2\log(x+1)] = 0$ $(x = -0.9)$

7. Riješi jednadžbe:

   a) $\frac{1}{5-\log x} + \frac{2}{1+\log x} = 1$ $(x_1 = 100;\ x_2 = 1000)$

   b) $\log(0.1x^2) \cdot \log\frac{10}{x} = -3$ $\left(x_1 = 100;\ x_2 = \frac{1}{\sqrt{10}}\right)$

   c) $2log_3^2(x-1) - \log_3(x-1)^2 = 4$ $\left(x_1 = 10;\ x_2 = \frac{4}{3}\right)$

8. Riješi sustav: $\begin{cases} 3^{x-2} \cdot 2^y = 54 \\ \log_2(x-y) = 2 \end{cases}$ $(x = 5; y = 1)$

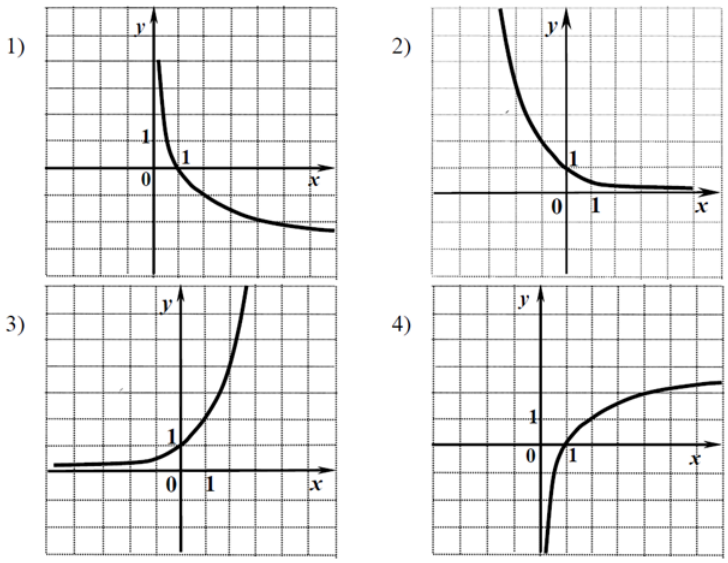
9. Riješi nejednadžbe:

   a) $\log_2(1 - 2\log_9 x) > 1$ $\left(x \in \langle 0; \frac{1}{3}\rangle\right)$

   b) $\log_{\frac{1}{3}} \frac{x-1}{2x+3} < 0$ $\left(x \in \langle -4; -\frac{3}{2}\rangle\right)$

10. Na kojoj je slici prikazan graf $f(x) = \log_2 x$

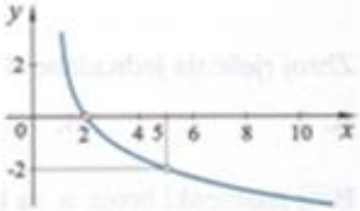
    (4)



1. Broj $\left(9^{1-\log_3 4}\right)^{-1.5}$ pripada intervalu:

A. $\langle -3,-1\rangle$ **B.** $\langle -1,1\rangle$ **C.** $\langle 1,3\rangle$ **D.** $\langle 3,5\rangle$

2. Na slici je prikazan graf funkcije:



A. $f(x)=\log_{\frac{1}{2}}(x-1)$
B. $f(x)=\log_{\frac{1}{3}}(x-1)$
C. $f(x)=\log_{\frac{1}{3}}x-1$
D. $f(x)=\log_2 x$

3. Ako je $f(x)=-9.5\cdot\ln(x-1.1)+5.2$, za koji je x $f(x)=2$?

A. $x\approx 2.5$ **B.** $x\approx -1.5$ **C.** $x=2$ **D.** $x\approx 3.5$

4. Nakon kraćenja razlomak $\dfrac{\log^3 x}{\log x^2}$ jednak je:

A. $\log^2 x$ **B.** $2\log^2\sqrt{x}$ **C.** $\log^2\sqrt{x}$ **D.** $\dfrac{\log^2 x}{2x}$

5. $\log(ab)-\log(bc)-\log(ca)=$

A. $\log(abc)$ **B.** $-2\log a$ **C.** $-2\log b$ **D.** $-2\log c$

6. Rješenje eksponencijalne jednadžbe $4\cdot 9^{1.5x-1}-27^{x-1}=33$ je:

A. $\dfrac{3}{4}$ **B.** 1 **C.** $\dfrac{4}{3}$ **D.** $-\dfrac{1}{2}$

7. Zbroj rješenja jednadžbe $2^{|x-1|}=16\cdot 4^{-0.5}$ jednak je:

A. -2 **B.** 0 **C.** 2 **D.** 4

8. Ako su x_1 i x_2 rješenja jednadžbe $\log^2 x-\log x^2=3$, tada je:

A. $x_1\cdot x_2=10^3$ **B.** $x_1\cdot x_2=1$
C. $x_1\cdot x_2=10^2$ **D.** $x_1\cdot x_2=10$

9. Ako je $\log 2=0.301$, $\log 3=0.477$, tada je jednakost $5^{x+2}=225$ ispunjena za:

A. $x\approx -1.115$ **B.** $x\approx 1.365$ **C.** $x\approx 2.2$ **D.** $x\approx -2.2$

10. Broj uređenih parova koji su realna rješenja sustava jednadžbi $\begin{cases} x^{-\frac{1}{2}}+y^{-\frac{1}{2}}=6 \\ \log_4 x+\log_4 y=-3 \end{cases}$ jednak je:

A. 0 **B.** 1 **C.** 2 **D.** 4

4. ispit

1	2	3	4	5	6	7	8	9	10
C	A	A	B	D	C	C	C	B	C

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